GRADUATING ESSAY

IMPLIED STANDARD DEVIATION OF STOCK INDEX OPTIONS

prepared for Prof. M. Brennan

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1. INTRODUCTION

The content of this paper can be broken into two parts. Firstly, it contains a documentation of weekly S&P500 futures option data that have been collected for the period of January 28, 1983 to the end of 1985. Since this data base will later be used for empirical tests of improved option valuation models, the documentation includes a detailed description of the way the data has been gathered, what plausability checks have been conducted, and how the information has been structured.

The derivation and application of the Implied Standard Deviation (ISD) for an option was originated by Latane and Rendleman (1976). The idea behind the Implied Standard Deviation is to use the current market price of the option in the Black-Scholes option valuation formula and to then solve for the unknown volatility σ of the futures price. The implicit standard deviation, therefore, estimates the volatility of the futures as it is expected by the market.

In the second part of this essay, a time series of such implied standard deviations for the S&P500 stock index options is computed. Qualitative factors are discussed that might have influenced the shape and the trend of the curve. Finally, a regression analysis is conducted to check whether the series can be modelled as a mean reverting process.

2. DOCUMENTATION OF DATA BASE

2.1 CONTENT

The data base contains data on both call and put options, the underlying S&P500 futures contract, and the yield on U.S. Treasury Bills (T-bills). It was gathered for the last trading day of each week (generally Friday) from the Wall Street Journal covering the period from January 28, 1983 to December 27, 1985.

The first records of the file, shown in Appendix 1, include a short description of the data organization. Appendix 2 shows which tables in the Wallstreet Journal were used to extract the information.

Options : Put and call option prices are listed for usually 3 maturity dates and up to 10 strike prices.

Futures : Closing prices for S&P500 futures are listed for up to 6 maturity dates which coincide with the maturity date of the option contract.

U.S. T-Bill Rate : Interest rates on U.S. T-Bills were gathered from the same source. The yield of the bill closest to the expiration date of the options and futures respectively was used.

Maturity : Options and the underlying futures contract expire four times a year on the third Saturday of March, June, September and December.

Time Scale : The time is expressed in weeks beginning with week 1 on January 28, 1983 and ending with week 153 on December 27th, 1985. (e.g. Saturday, Dec 22, 1984 is equal to week 100)

2.1 PLAUSABILITY CHECKS

A host of plausability checks have been conducted to minimize the number of typing errors made while collecting the data.

Manual Spot Checks : Manual comparison of file content and figures given in the Wallstreet Journal was conducted for about ten percent of the data.

Plausability Programs : These programs were written to check general plausability conditions. Appendix 3 contains an excerpt of plausabilities tested.

The tests revealed the following general problems with the data :

- Option prices very far in-the-money and out-of-money do not seem reliable. Because these options are not traded heavily, their prices may not be up-to-date. Thus, even though the listed prices may have been paid at the beginning of the day, the underlying futures price has sometimes moved considerably during the trading session. An indication for these differences in timing were given by the great number of arbitrage opportunities for options far in-the-money .

- The reporting of low value options is not consistent. Even for options that obviously have no more value, prices up to 5 cents were given.
- Missing data posed problems. Options far away from expiration are not traded heavily. Thus, often no prices were available. Not all the T-bill rates corresponding to the futures contracts were given. This happened, however, only rarely for futures with options.
- The readability of the microfiches was often unsatisfactory. Whilst the magnitude of a figure can be estimated from the context, this is hardly possible for the digits behind the decimal point. Yet the number and seriousness of errors from this source is probably limited.

Based upon these findings, a Fortran Subroutine was written that filters the data and stores it into arrays to be used for further processing. The program is documented and listed in Appendices 4.1 thru 4.3.

The program eliminates

- option prices of less than 10 cents,
- option prices without a corresponding futures price or interest rate, and
- options with strike prices outside the interval futures price of ± $10.

There are still a few suspect options that pass these filters. A list of these values is included in Appendix 5. Most of them violate the arbitrage condition, but usually only by a few cents.

3. TIME SERIES IMPLIED STANDARD DEVIATIONS

3.1. DERIVIATION

The computation of the Implied Standard Deviation (ISD) applies Black's (1976) model for pricing European futures options. This formula is equivalent to Black-Scholes' (1973) stock option valuation formula without dividend, but it uses the spot price (present value of the futures price) instead of the stock price.

Black's Futures Option Pricing Model :

$$C = +\{e^{**}(-r\tau)[FN(+d1)-EN(+d2)]\} \quad (1)$$

$$P = -\{e^{**}(-r\tau)[FN(-d1)-EN(-d2)]\} \quad (2)$$

with

$$d1 = [\ln(F/E) + (\sigma^2/2)\tau]/\sigma\sqrt{\tau}$$
$$d2 = d1 - \sigma\sqrt{\tau} \quad (3)$$

where

C and P	are the model prices of the futures put and call options,
F	the underlying futures price,
E	the exercise price of the option,

τ the option's remaining time to maturity in terms of years,

r the risk-free rate annualized, and

σ the instantaneous standard deviation of returns of the underlying futures contract for the remaining life of the option.

N(.) is the cummulative normal density function.

The fact that S&P500 Stock Index Options can be exercised early (American option) has been neglected because differences are usually small (Jarrow and Rudd, 1982).

Unfortunately, these formulas cannot be inverted to calculate the ISD σ directly. So numerical iteration procedures must be used. The listing of the Fortran Subroutine employed to find the ISD for the market option price can be found in Appendix 6. The formula is first solved for an initial value of the variance of 0.015. Then the program approaches the market value of the option, adjusting this variance in increasingly finer steps, until the absolute value of the difference between model and market option price is less than 0.001. Problems in the algorithm arise for small standard deviations when the estimate becomes zero. In this case, the estimate is adjusted to a figure slightly higher than zero. In some cases, however, the market price actually implied a standard deviation of zero. These options coincide with the few suspect ones identified earlier (Appendix 5). However, since they are so few in number, they won't mar the general trend of the time series.

For each of the 153 weekly data sets, ISDs for all the put and call options with strike prices within the ± \$10 range of futures price, were calculated. Given that the ISDs for the options of a certain week differed considerably, the question as to which should be the representative ISD for the week arises. The obvious solution would be to compute the arithmetic mean of all ISDs per week. Yet Latane and Rendleman (1976) labelled such a weighting scheme "unreasonable". They found out that options very sensitive to a change in σ seem to give the best prediction of future volatility. These options should therefore be weighted heavier than others. Basing upon these findings, D.P. Chiras and S. Monaster (1977) suggested to weight ISDs according to the sensitivity of the dollar price change for options relative to the incremental change in the ISDs. They argued that a rational investor measures returns at a ratio of dollar price change relative to the size of the investment. In order to be consistent with a rational measure of returns, the price elasticity of the options with respect to their ISDs should be considered.

This is the equation used to obtain the Weighted Implied Standard Deviation (WISD) for the futures options at each observation date :

$$WISD = \frac{\sum_{j=1}^{N} \sigma_j \, (\partial C_j/\partial \sigma_j)\,(\sigma_j/C_j)}{\sum_{j=1}^{N} (\partial C_j/\partial \sigma_j)\,(\sigma_j/C_j)} \qquad (4)$$

where

N is the number of options recorded for the observation date,

WISD the weighted implied standard deviation for the observation date,

σ_j the implied standard deviation of option j, and

$\frac{\partial C_j}{\partial \sigma_j} \cdot \frac{\sigma_j}{C_j}$ the price elasticity of option j with respect to to its implied standard deviation.

In the context of Black's Option Pricing Model (OPM), the partial with respect to the standard deviation can be expressed explicitly as

$$\partial C/\partial \sigma = \sqrt{\tau}\, F[e^{**}(-r\tau)](1/\sqrt{2\pi})[e^{**}(-d1^2/2)] \qquad (5)$$

where the variables are explained as in equations (1)-(3)

(Source: Jarrow and Rudd (1983) employing spot price instead of stock price)

Appendix 7, finally, contains the plot of these WISDs for the 153-week observation period.

3.2 INTERPRETATION

Deriving the partial differential equation in Black's OPM, one has to assume a constant standard deviation of the futures return over time. Yet as can be seen in Appendix 7, the implied volatilities fluctuate considerably over time. They start off as high as 21% and reach slumps as low as 10%. The WISD for the observation period averages 13.5% with a standard deviation of 2.4%. If the assumption of constant volatilities held, options would be systematically over- or underpriced respectively over long periods of time. This is not very likely in an approximately efficient market. Rather, it must be suspected that the expectation of the future volatility is changing constantly.

Sudden movements of the stock prices seem to alter these expectations dramatically. At the beginning of August 1984 (week 81) the implied standard deviation started soaring from a level of 12% up to 17%. A look at stock market data reveals that this peak was fueled by an unprecedented rally (Appendix 8). The Dow Jones Industrials Average took record jumps and the N.Y.S.E. volume became so huge that the Wallstreet Journal had to squeeze the scales on its volume charts.

Given the volatiliy of the implied standard deviation, it becomes clear that efforts to improve the Black's OPM have to focus on a more realistic modelling of this parameter. One possibility is to use a mean reverting process. This would imply that the farther the WISDs are pushed away from an equilibrium level, the faster they would tend to approach this level again thereafter. The hypothesis is that this asymptotic approach follows an exponential function :

$$\sigma(t) = m-(m-\sigma_0)[e**(-at)] \qquad (6)$$

This equation is the solution of the following differential equation :

$$\Delta\sigma/\Delta t = a(m-\sigma) \qquad (7)$$

where

$\Delta\sigma/\Delta t$ is the change in the implied standard deviation per incremental time unit, in this case a week,

$\sigma(t)$	is the weighted implied standard deviation for the observation date (WISD),
m	the equilibrium level of WISD,
a	the measure of how fast this equilibrium is approached, and
σ_0	the WISD at time zero.

A regression analysis was conducted to check whether there is, in fact, a linear link between the change in the WISD and the WISD itself. The chart in Appendix 9 plots the WISD at week t against the change in WISD from week t to week t+1. The shape of the "cloud" points to a significant correlation between the two variables. As expected the changes seem to be rather positive when the WISDs are low and negative when the WISD is above average.

In fact, the coefficient and the constant of the regression line turn out to be significantly different from zero (p-value < 0.1%). Further, 'm' is estimated as 13.1% with a standard error of 4.1%, and the best estimate of 'a' is 0.096 with a standard deviation of 0.012. This 'a'-value would imply that after about 7 weeks an initial deviation from the equilibrium level has been reduced by half (half-life 7.2 weeks).

Unfortunately the confidence intervals for the two parameters are quite broad so that the a future ISDs cannot be predicted very precisely. A mean reverting process is, therfore, although a better way to model the time series, certainly not the perfect method.

REFERENCES

Black,F. and Scholes,M. (1973); "The pricing of options and corporate liabilities"; J. Political Economy, May-June, 637-658

Black,F. (1976); "The pricing of commodity contracts"; J. Financial Economics, 3 ,167-179

Jarrow,R. and Rudd,A.(1983); Option Pricing ; Homewood, Ill.; Richard D. Irwin, Inc.

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THIS FILE CONTAINS PRICES OF FUTURES AND UNDERLYING OPTIONS
FOR THE S&P500 STOCKINDEX TAKEN FOR THE PERIOD OF JAN 28, 1983 TO
DEC 27,1985 EACH FRIDAY.

THE FILE IS ORGANIZED THE FOLLOWING WAY :

4 TYPES OF RECORDS IDENTIFIED BY RECORDTYPE CHARACTER IN THE
FIRST COLUMN.
TYPE 1 - INITIAL DATERECORD
  COL 2-4   : WEEK (COUNTED BEGINNING 1/28/83)
  COL 11-16 : MMDDYY
  COL 17-30 : COMMENTS
  COL 30-33 : NUMBER OF TYPE 2 RECORDS THAT FOLLOW

TYPE 2 - FUTURE RECORD
  COL 2-4   : WEEK
  COL 5-7   : EXPIRATION WEEK OF THE FUTURE
  COL 11-15 : PRICE OF FUTURE (F5.2)
  COL 16-20 : T-BILL RATE AT EXPIRATION (THE CLOSEST ONE)
  COL 21-30 : COMMENTS; USED IF DATA ARE PECULIAR
  COL 30-33 : NUMBER OF TYPE 3 RECORDS THAT FOLLOW

TYPE 3 - OPTION RECORD
  COL 2-4   : WEEK
  COL 5-7   : EXPIRATION WEEK
  COL 8-10  : STRIKE PRICE
  COL 11-15 : PRICE CALL (F5.2)
  COL 16-20 : PRICE PUT  ( "  )
  COL 21-30 : COMMENTS

TYPE 9 - COMMENTS COL 1-70

THE RECORDS ARE SORTED IN THE FOLLOWING ORDER :
  WEEK, EXPIRATION WEEK, STRIKE PRICE

PROBLEMS WITH THE DATA (OFTEN ALLEGED IN COMMENT COLUMNS):

- OPTION PRICES LOWER THAN $0.10 SHOULD BE IGNORED (UNRELIABLE)

- FOR FUTURES BEFORE EXPIRATION DATE NO T-BILL RATE AVAILABLE
  NO FUTURE PRICE FOR OPTIONS AT EXPIRATION
  CONSEQUENCE : IGNORE OPTION PRICES IF FPRICE OR RF = 0


 VANCOUVER, MAY 30, 1986        KURT HESS
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FUTURES OPTIONS

Friday, October 25, 1985. = week 144

–AGRICULTURAL–

ORN (CBT) 5,000 bu.; cents per bu.

ike e	Calls–Settle Dec-C	Mar-C	May-C	Puts–Settle Dec-P	Mar-P	May-P
	20½	....	....	⅛	¼	....
	10½	25	....	¼	½	1
	3¼		..			2
	3/8					
114			2.05	5.95	5.85	5.85

Est. vol. 401; Thur.: vol. 75 calls, 100 puts
Open interest Thurs.;4,217 calls, 3,294 puts

S&P 500 STOCK INDEX (CME) $500 times premium.

Strike Price	Calls – Settle Dec-C	Mar-C	Jn-C	Puts – Settle Dec-P	Mar-P	Jun-P
175	12.50	....	16.45	0.30	1.10	1.65
180	8.20	10.85	....	0.90	2.05	2.70
185	4.60	7.40	....	2.20	3.45	4.20
190	2.20	4.85	6.80	4.75	5.65	6.30
195	0.85	2.90	4.70	8.40	8.65	9.00
200	0.30	1.70	3.15	12.75	....	12.20

Est. vol. 6,715; Thur.; vol. 1,894 calls; 943 puts
Open interest Thurs.; 19,045 calls; 20,993 puts

S&P 500 FUTURES INDEX (CME) 500 times index

Dec	187.65	187.90	186.90	187.40	–	.70	200.85	175.40	58,608
Mar86	189.25	189.50	188.55	189.05	–	.65	203.75	182.30	4,247
June	190.50	190.75	190.00	190.50	–	.55	206.60	183.90	177

Est vol 46,341; vol Thurs 46,314; open int 63,109, +1,790.

S&P 500 STOCK INDEX (Prelim.)
188.15 188.51 187.32 187.52 – .98 ... .

```
1144   0    0102585              3
2144152     018740      741      6
3144152175   1250       030
3144152180    820       090
3144152185    460       220
3144152190    220       475
3144152195    085       840
3144152200    030      1275
2144165     018905      756      6
3144165175              110
3144165180   1085       205
3144165185    740       345
3144165190    480       565
3144165195    290       865
3144165200    170
2144178     019050      785      6
3144178175   1645       165
3144178180              270
3144178185              420
3144178190    680       630
3144178195    470       900
3144178200    315      1220
```

U.S. Treas. Bills

Mat. date	Bid	Asked Discount	Yield	Mat. date	Bid	Asked Discount	Yield
-1985-				-1986-			
				2-13	7.31	7.27	7.53
10-31	6.60	6.50	6.59	2-20	7.32	7.28	7.56
11- 7	7.09	7.05	7.16	2-27	7.32	7.28	7.57
11-14	7.26	7.22	7.34	3- 6	7.33	7.29	7.59
11-21	7.15	7.09	7.22	3-13	7.34	7.30	7.61
11-29	7.22	7.18	7.33	3-20	7.28	7.24	7.56
12- 5	7.23	7.19	7.34	3-27	7.29	7.25	7.58
12-12	7.26	7.22	7.39	4- 3	7.42	7.38	7.73
12-19	7.27	7.23	7.41	4-10	7.42	7.38	7.74
12-26	7.29	7.25	7.44	4-17	7.44	7.40	7.77
-1986-				4-86	7.42	7.40	7.79
1- 2	7.26	7.24	7.44	5-15	7.45	7.41	7.81
1- 9	7.28	7.24	7.45	6-12	7.46	7.44	7.85
1-16	7.28	7.24	7.46	7-10	7.50	7.46	7.90
1-23	7.24	7.22	7.45	8- 7	7.52	7.48	7.94
1-30	7.26	7.22	7.46	9- 4	7.53	7.49	7.99
2- 6	7.29	7.25	7.50	10- 2	7.52	7.48	8.01

APPENDIX 3

PLAUSABILITY CHECKS (EXCERPT)

- Week must grow exactly by one from one type-1 record to the next

- Expiration week may take only a certain set of values (options and futures expire only 4 times a year) and must grow from one maturity to the other.

- Futures price and riskfree rate should generally increase as the maturity becomes more distant. Even though for the riskfree rate a number of slightly declining values have been detected, the figures seemed plausible.

- While the value of the call option must decrease with increasing exercise price, the the value of the put must increase.

- Arbitrage Conditions :

  CALLS : Futures Price - Exercise Price > Call Price

  PUTS : Exercise Price - Futures Price > Put Price

```
C THIS SUBROUTINE READS THE OPTION DATA AND WRITES THEM
C IN DATA ARRAYS FOR SUBSEQUENT PROCESSING
C
C DESCRIPTION OF VARIABLES
C -----------------------
C
C  SUBSCRIPTS :   J     WEEK (1 .. 153)
C
C                 K  =  1   1ST OPTION CLOSEST TO MATURITY
C                       2   2ND OPTION    "
C                       3   3RD OPTION    "
C                       4   4TH OPTION    "
C                       5   1ST OPTION INTERMEDIATE TO MATURITY
C                       6   2ND OPTION    "
C                       7   3RD OPTION    "
C                       8   4TH OPTION    "
C                       9   1ST OPTION  MOST DISTANT TO MATURITY
C                      10   2ND OPTION    "
C                      11   3RD OPTION    "
C                      12   4TH OPTION    "
C
C                      THE 4 OPTIONS FOR EACH MATURITY ARE
C                      TAKEN FROM THE INTERVAL FUTURES PRICE
C                      PLUS/MINUS 10
C
C                       1ST  2ND  3RD  4TH
C                   I....I....I....I....I...
C                   -10       E        +10
C                    <.........X.........>
C
C                      IF THERE ARE 5 EXERCISE PRICES IN THE
C                      INTERVAL (FUTURES PRICE EQUALS EXERCISE
C                      PRICE), THE LAST ONE IS DROPPED.
C
C
C                 L    OPTION CODE    1 = CALL
C                                     2 = PUT
C
C
C   C(J,K,L)           OPTION PRICE
C   E(J,K,L)           EXERCISE PRICE
C   F(J,K,L)           FUTURES PRICE
C   M(J,K,L)           WEEKS TO MATURITY
C   RF(J,K,L)          RISKFREE RATE
C
C
C   IN CASES WHERE DATA ARE NOT AVAILABLE OR WHEN THEY DO
C   NOT SEEM CLEAN, THE ARRAY ELEMENT IS LEFT WITH ITS
C   INITIAL VALUE OF -1.
C
C
C
      DIMENSION C(153,12,2),E(153,12,2),F(153,12,2),
     *          M(153,12,2),RF(153,12,2)
      CHARACTER REC*70
      INTEGER   TYPE,EXWEEK
C
C      INITIALIZE ARRAYS
C
      DO 10 J=1,153
```

```
      DO 11 K=1,12
        DO 12 L=1,2
          C(J,K,L)=-1.
          E(J,K,L)=-1.
          F(J,K,L)=-1.
          M(J,K,L)=-1
          RF(J,K,L)=-1.
 12     CONTINUE
 11   CONTINUE
 10 CONTINUE
C
C      READING FIRST RECORD OF WEEK (RECORD TYPE 1)
C
 20 READ (5,100,END=50) TYPE,REC
100 FORMAT (I1,A70)
    IF (TYPE.EQ.1) GO TO 30
    IF (TYPE.EQ.9) GO TO 20
    WRITE (6,200) TYPE,REC
200 FORMAT ('*** DATA OR FILE ERROR ***',I1,A69)
    GO TO 60
C
C      FILLING WEEKLY DATA IN ARRAYS
C
 30 READ(REC,101) NUM2
101 FORMAT (29X,I3)
    I=1
    IF (NUM2.EQ.0) GO TO 40
    DO 40 I1=1,NUM2
      READ (5,102) J,EXWEEK,FUT,R,NUM3
102   FORMAT(1X,I3,I3,3X,F5.2,F5.4,10X,I3)
      IF (NUM3.EQ.0) GO TO 41
      DO 41 I2=1,NUM3
        READ(5,103) EX,CALL,PUT
103     FORMAT (1X,3X,3X,F3.0,F5.2,F5.2)
        IF (FUT.EQ.0.OR.R.EQ.0) GO TO 41
        CALL POS(FUT,EX,N)
        IF (I.GT.3)            GO TO 41
        IF (N.EQ.0)            GO TO 41
        K=4*(I-1)+N
        IF (CALL.LT.0.1)       GO TO 42
        C(J,K,1) = CALL
        E(J,K,1) = EX
        F(J,K,1) = FUT
        RF(J,K,1) = R
        M(J,K,1) = EXWEEK-J
 42     CONTINUE
        IF (PUT.LT.0.1) GO TO 43
        C(J,K,2) = PUT
        E(J,K,2) = EX
        F(J,K,2) = FUT
        RF(J,K,2) = R
        M(J,K,2) = EXWEEK-J
 43     CONTINUE
        N=N+1
 41   CONTINUE
      I=I+1
 40 CONTINUE
    GO TO 20
C
C      PRINTING OF ARRAYS AND PROGRAM END
```

```
C
   50 DO 60 J=1,153
         DO 61 K=1,12
           WRITE (6,210) J,K,M(J,K,1),F(J,K,1),RF(J,K,1),
     *                   E(J,K,1),C(J,K,1)
  210      FORMAT (I3,I3,I3,F7.2,F7.4,F7.2,'  C',F7.2)
           WRITE (6,211) J,K,M(J,K,2),F(J,K,2),RF(J,K,2),
     *                   E(J,K,2),C(J,K,2)
  211      FORMAT (I3,I3,I3,F7.2,F7.4,F7.2,'  P',F7.2)
   61    CONTINUE
   60 CONTINUE
      WRITE (6,220)
  220 FORMAT ('*** PROGRAM END')
      STOP
      END
C
C
C      SUBROUTINE TO DETERMINE THE POSITION N OF THE OPTION
C      PRICE IN THE INTERVAL FUTURES PRICE PLUS/MINUS 10
C
      SUBROUTINE POS(FUT,EX,N)
      N=0
      IF ((EX-FUT).LE.10.AND.(EX-FUT).GT.5)      N=4
      IF ((EX-FUT).LE.5.AND.(EX-FUT).GT.0)       N=3
      IF ((EX-FUT).LE.0.AND.(EX-FUT).GT.-5)      N=2
      IF ((EX-FUT).LE.-5.AND.(EX-FUT).GT.-10)    N=1
      END
```

APPENDIX 5

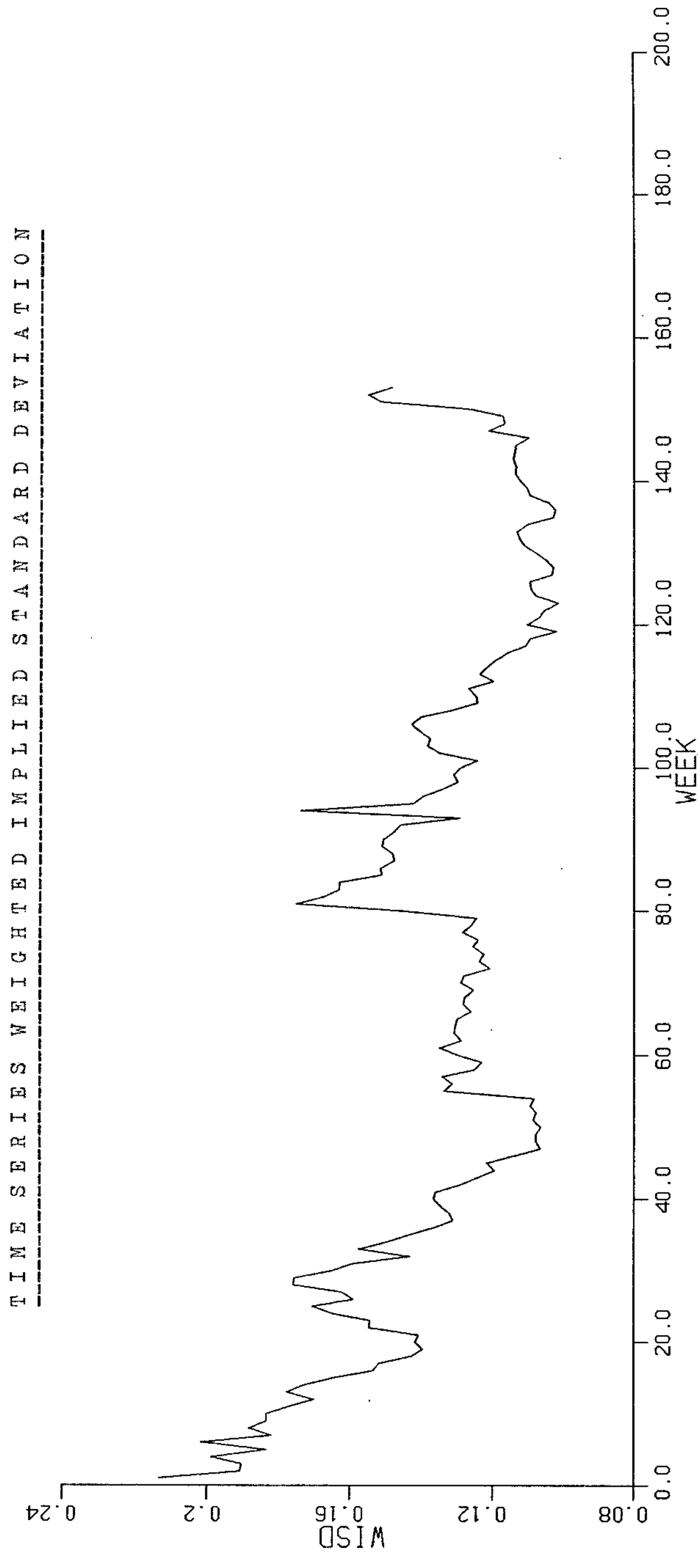
SUSPECT OPTIONS

Based upon several plausability checks, the following options have been found suspect. Options filtered out anyway by the read subroutine in Appendix 4 are not included.

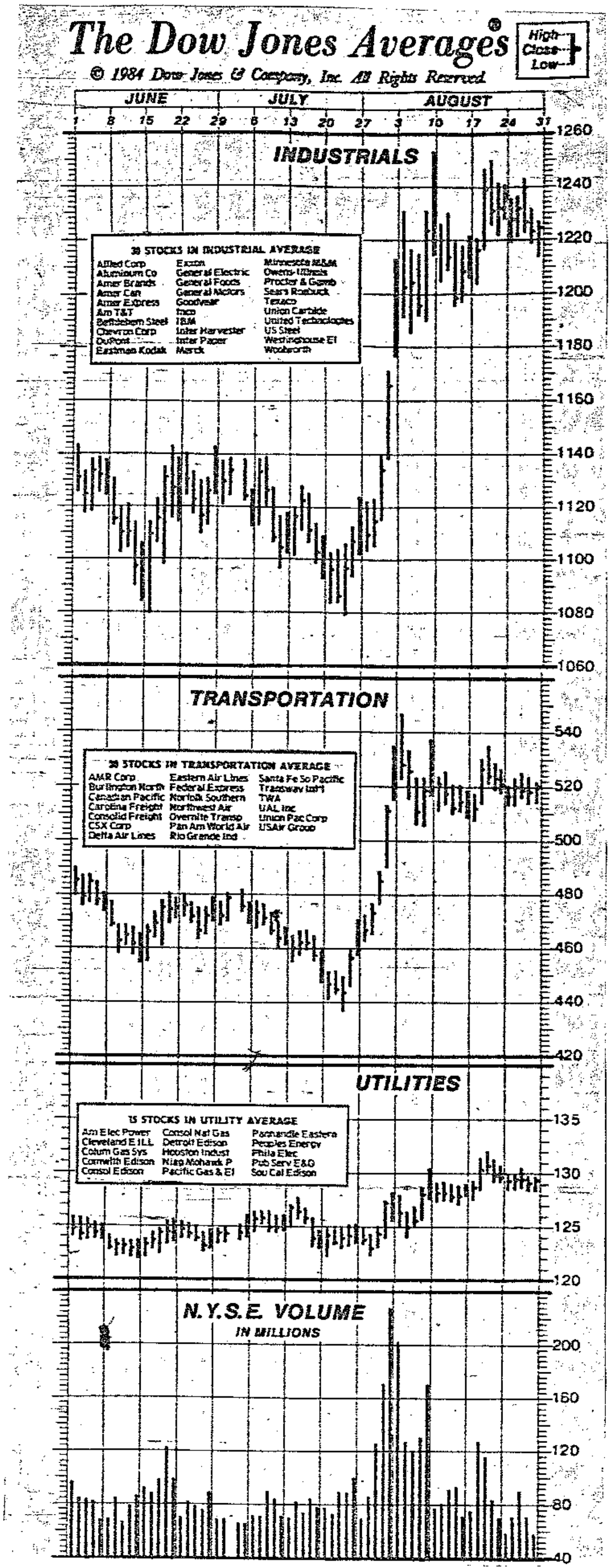
WEEK/EXWEEK	STRIKE	FUTURE	OPTION PRICE		ERROR
7/ 8	160	151.25	8.70	PUT	Arbitrage $0.05
18/ 21	175	165.60	4.90	PUT	Arbitrage $4.50
33/ 34	160	167.05	7.00	CALL	Arbitrage $0.05
56/ 60	165	156.30	8.30	PUT	Arbitrage $0.40
58/ 60	150	159.45	9.40	CALL	Arbitrage $0.05
77/100	160	155.20	6.05	CALL	CALL growing
79/ 87	160	151.25	7.80	PUT	Arbitrage $0.95
93/100	165	170.05	3.50	CALL	Arbitrage $1.55
98/100	155	162.50	7.45	CALL	Arbitrage $0.05
120/152	190	192.45	1.75	CALL	Arbitrage $0.70
137/139	185	188.50	3.25	CALL	Arbitrage $0.25

```
C       SUBROUTINE TO CALCULATE THE IMPLIED STANDARD VARIANCE
C       OF THE SPOT PRICE FOR A CALL OPTION
C                                ****
C
C.      C :      EXERCISE PRICE
C       DELTA:   APPROACH STEP
C       EXWEEK:  EXPIRATION WEEK
C       H :      ARGUMENT NORMAL DENSITY FUNCTION
C       I :      LOOP COUNTER
C       R :      RISKFREE RATE
C       S :      SPOTPRICE (PRESENT VALUE OF FUTURES PRICE)
C       SWITCH:  SWITCH TO GIVE THE DIRECTION OF SEARCH
C                PROCESS
C       TE:     (TIME TO EXPIRATION IN YEARS)
C       VAL:     OPTION VALUE FROM B-S MODEL
C       V2:      IMPLIED VARIANCE RATE
C       WEEK:    WEEK
C       WV:      OPTION MARKET PRICE
C
        SUBROUTINE ISD(WEEK,EXWEEK,TE,S,R,C,WV,V2,H)
C
C
C
        INTEGER SWITCH
        V2=0.015
        DELTA=0.005
        SWITCH=0
        I=0
      1 IF (I.GT.100) GO TO 10
        I=I+1
        CALL BS(V2,R,S,TE,C,VAL,H)
        IF (ABS(WV-VAL).LT.0.001) GO TO 10
        IF (WV-VAL) 5,10,6
      5 IF (SWITCH.EQ.1) DELTA=DELTA/4
        V2=V2-DELTA
        SWITCH=-1
        IF (V2.GT.0) GO TO 1
        V2=+0.00000000001
        GO TO 1
      6 IF (SWITCH.EQ.-1) DELTA=DELTA/4
        V2=V2+DELTA
        SWITCH=+1
        GO TO 1
     10 CONTINUE
        RETURN
        END
```

Appendix 7



TIME SERIES WEIGHTED IMPLIED STANDARD DEVIATION
WISD
0.08
0.12
0.16
0.2
0.24
WEEK
0.0
20.0
40.0
60.0
80.0
100.0
120.0
140.0
160.0
180.0
200.0



Record Day for Big Board Volum
Is Fueled by Heavy Block Trading

By Pamela Sebastian and Tim Metz

A record number of large trades on the New York Stock Exchange yesterday helped stoke trading to its best one-day volume ever.

Trades of blocks of more than 10,000 shares hit a record 3,579 on the Big Board, breaking the previous high of 3,151 set Jan. 5, and helping push total volume to a record of 172,830,000 shares. The Dow Jones Industrial Average rose 31.47 points to 1166.08.

(Wallstreet Journal, Aug 3rd, 1984)

Institutions Lead Way as Volume
On Big Board Sets Another Recor

By Pamela Sebastian

Institutions kept buying big Friday, topping off a record trading week with the first 200-million-share day on the New York Stock Exchange.

Back-to-back record volume on Thursday and Friday drove weekly volume on the Big Board to a record 696,162,760, besting the previous peak of 592,445,661 set the week of Oct. 15, 1982. Year-earlier volume for the same summer week was 400,860,-440.

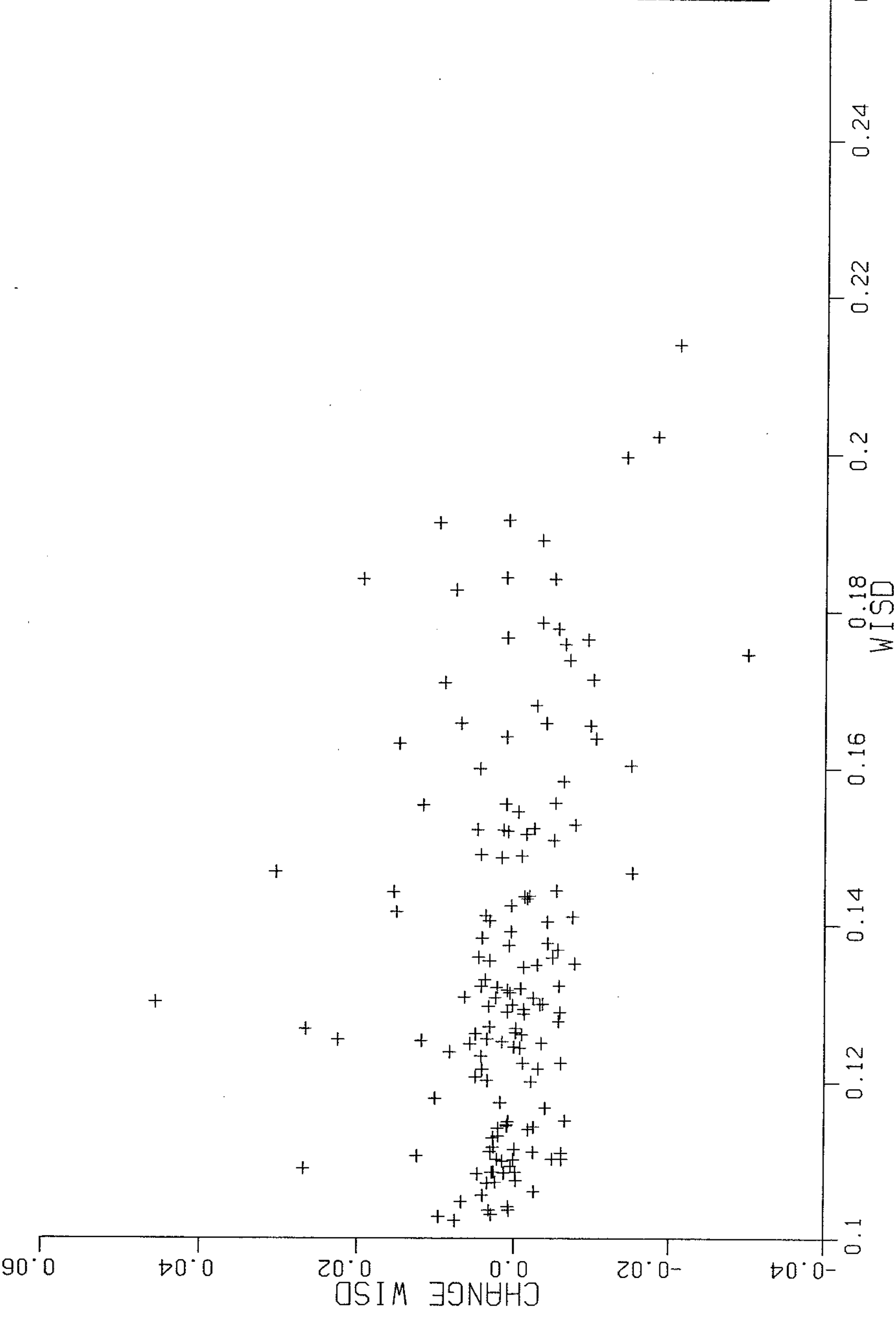
For all of last week, the Dow Jones Industrial Average jumped 87.46 points, or 7.8%. It was the biggest weekly point-move for the Dow ever, shattering the 81.24-point gain the week ended Aug. 20, 1982.

(Wallstreet Journal, Aug 6th, 1984)

Year's Worst Performing Stocks
Rebound to Lead the August Rally

(Wallstreet Journal, Aug 13th, 1984)

Appendix 9


WISD
CHANGE WISD
0.1
0.12
0.14
0.16
0.18
0.2
0.22
0.24
0.26
0.06
0.04
0.02
0.0
-0.02
-0.04