The Theory of the Banking Firm

Inspired by
Matthews & Thompson, 2008, 'The Economics of Banking', 2nd edition, Wiley, chapter 6
Freixas & Rochet, 1997, Microeconomics of Banking, MIT Press, chapter 3

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Multiplier approach

Bank balance sheet

Assets	Liabilities
Loans (L)	Deposits (D)
Reserves (R)	

Money / credit multipliers

- A change in the monetary base has a direct impact on money and credit in the system
- The multiplier describes the transmission of changes in monetary base on loans (credit) and deposits (savings) in the system

Money Multiplier

- Base money (H) and broad money (M) definition

$$H = C + R$$

$$M = C + D$$

with

C: currency in circulation by non-bank public

R: bank reserves

D: bank deposit

Money Multiplier

- Money multiplier (m)

$$m=\frac{M}{H}=\frac{C+D}{C+R}=\frac{C/D+1}{C/D+R/D}=\frac{c+1}{c+k}$$

$c=C/D$ c: currency-deposit ratio

$k=R/D$ k: reserve-deposit ratio

Credit / deposit multiplier

Derive credit multiplier

Credit multiplier

$$R=kD$$

$$L=(1-k)D \quad \div H=C+R$$

$$\frac{L}{H}=\frac{(1-k)D}{C+R}\Rightarrow$$

$$\Delta L=\frac{1-k}{C/D+R/D}\Delta H=\frac{1-k}{c+k}\Delta H$$

Credit / deposit multiplier

Derive deposit multiplier

Deposit multiplier

$$\frac{D}{H}=\frac{D}{C+R}\Rightarrow$$

$$\Delta D=\frac{1}{C/D+R/D}\Delta H=\frac{k}{c+k}\Delta H$$

Multipliers criticism

- Model sees banks as passive agents, i.e. behavioural component
- Assumes fixed c and k but …
  - c is a choice variable by non-bank public affected by deposit rate r_D (and other factors)
  - k is a choice variable for the bank (in the absence of binding requirement)
- Supply of base money H not exogenous but supplied by central bank driven by demand.

Perfect competition

Perfect competition model Bank balance sheet

Assets	Liabilities
Loans (L)	
Liquid assets (T)	Deposits (D)
Reserves (R)	

Liquid assets (T) are short-term, riskfree assets such as treasury bill, short-term interbank lending at rate r_T.

Loan and deposit rates r_L and r_D are constant as far as the individual bank is concerned.

Bank reserve R (not interest-bearing) with R = kD; k: reserve-deposit ratio

Perfect competition model
Profit function

$$\pi = r_L L + r_T T - r_D D - \underbrace{l - s - c}$$

$$\pi = r_L L + r_T T - r_D D - C(D,L)$$

Where

L, D, T, r_T, r_L, r_D as defined before; l: cost of illiquidity,
s: cost due to default, c: real resource cost
C(D,L) cost of generating D, L with $\partial C / \partial L \geq 0, \partial C / \partial D \geq 0$

Perfect competition model
Maximizing profit function

$$\partial \pi / \partial L = r_L - r_t - \partial C / \partial L (D,L) = 0$$

$$\partial \pi / \partial D = r_t(1-k) - r_D - \partial C / \partial D (D,L) = 0$$

thus

$$r_L = r_T + \partial C / \partial L$$

$$r_D = r_T(1-k) + \partial C / \partial D$$

$$r_L = r_D + k r_t + \partial C / \partial L + \partial C / \partial D$$

Perfect competition model

- A bank will thus adjust its volume of loans and deposits such that intermediation margins $r_L - r_T$, resp. r_T (1-k) – r_D equal marginal management costs
- No cross effects if costs are separable, i.e. $\frac{\partial^2 C}{\partial L \partial D} = 0$

Perfect competition model

- Cross-effects are likely, however, e.g. through shared fixed cost of operations
- Assume $\frac{\partial^2 C}{\partial L \partial D} < 0$ which means an increase in r_L (resp. r_D) entails an increase in deposits, respectively loans
- Economies of scope if increase L (D) decreases marginal cost of D (L). Helps explains existence of universal banks.

Competitive equilibrium

- In equilibrium, with n different banks …
  - the cumulative loan supply of these banks must equal the investment demand of firms in the system (I(r_L) function)
  - the cumulative savings S(r_L) of households is sum of deposits and treasury bills (B)

$$I(r_L) = \sum_{n=1}^{N} L^n(r_L, r_D, r_T) \quad \text{Loans market}$$

$$S(r_D) = B + \sum_{n=1}^{N} D^n(r_L, r_D, r_T) \quad \text{Savings market}$$

Competitive equilibrium (2)

- In equilibrium, with n different banks …
  - the aggregate position of all banks in the interbank market is zero (determines interest rate in interbank market)

Interbank market

$$\sum_{n=1}^{N} L^n(r_L, r_D, r_T) = (1-k) \sum_{n=1}^{N} D^n(r_L, r_D, r_T)$$

k: reserve-deposit ratio

Determination of loan rate Simplified numerical example

- Numerical example to illustrate determination of loan rate r_L
- Assumes functional form for loan cost function of competitive bank (not correlated to cost of deposit production)
- Given T (liquid assets) and market rate r_T, premium ($r_L - r_T$) equals marginal loan cost / minimum average loan cost

Determination of loan rate Simplified numerical example

Cubic cost function with fixed cost term FC; γ_L is a constant

$$C(L) = FC + \gamma_L L^3$$

$$\partial C / \partial L = 3\gamma_L L^2$$

Average cost per $ of L:

$$C/L = \frac{FC}{L} + \gamma_L L^2$$

Marginal average cost:

$$\frac{\partial C/L}{\partial L} = \frac{-FC}{L^2} + 2\gamma_L L$$

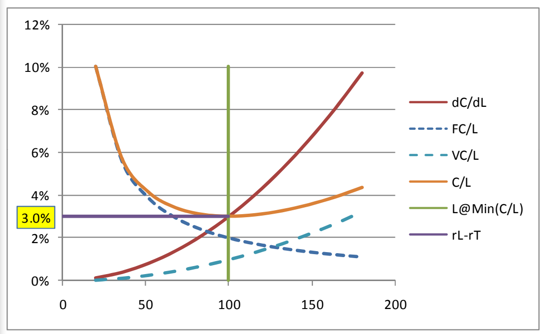
L at marginal average cost = 0:

$$L @ \min\left(\frac{\partial C/L}{\partial L}\right) = \left(\frac{FC}{2\gamma_L}\right)^{1/3}$$

Numerical example

	A	B	C	D	E
1	**Illustrating competitive model**				
2	L	Loans	$ 50.00		
3	gammaL	1000000	0.000001	=1/B3	
4	FC	Fixed cost	$ 2.00		
5	VC	Variable cost	$ 0.125	=C3*C2^3	
6	C(L)	Total cost	$ 2.125	=C5+C4	
7					
8	dC/dL	Marginal cost	0.7500%	=3*C3*C2^2	
9	FC/L	Average fixed cost	4.0000%	=C4/C2	
10	VC/L	Average var. cost	0.2500%	=C5/C2	
11	C/L	Average total cost	4.2500%	=C6/C2	
12					
13	L@Min(C/L)		$ 100.00	=(C4/C3/2)^(1/3)	
14	(C/L)min		3.0%	=(C4+C3*C13^3)/C13	
15					
16	T	Liquid assets	$ 10.00		
17	rT		4.00%		
18	rL	=rT- (C/L)min	7.00%	=C17+C14	
19					
20		Interest income	$ 3.90	=C18*C2+C17*C16	
21	π(L)	Profit	$ 1.78	=C20-C6	

Numerical example



Monopolistic bank

- Based on Klein (1971) and Monti (1972) models
- The monopoly bank faces …
  - a downward-sloping demand for loans
  - an upward-sloping demand for deposits

Balance sheet: $L + T + R = D \qquad R = kD$

$$L^{Demand} = L(r_L)\, with\ \partial L/\partial r_L < 0$$

$$D^{Demand} = D(r_D)\, with\ \partial D/\partial r_D > 0$$

Monopolistic bank

- Scale and allocation decision. Volume of deposits (D) determines scale.
- Market for bills (T) is competitive at r_T (bank is price taker)
- Bank maximizes (reserves w/o interest):

$$\pi = r_L L(r_L) + r_T\left(D(1-k) - L\right) - r_D D(r_D) - C(D, L)$$

with $T = D(1-k) - L$

Monopolistic bank

- Monopolist can set price or quantity demanded but not both.
- Maximizing profit π if bank sets rate of interest r_L and r_D:

$$\partial\pi/\partial r_L = L + \left(\partial L/\partial r_L\right) r_L - \left(\partial L/\partial r_L\right) r_T - \left(\partial C/\partial r_L\right) = 0$$

$$\partial\pi/\partial r_D = (1-k)\left(\partial D/\partial r_D\right) r_T - D - \left(\partial D/\partial r_D\right) r_D - \left(\partial C/\partial r_D\right) = 0$$

Monopolistic bank

- Derive optimal r_{L*} and r_{D*} for given L and D, slope of demand curve, riskfree rate and management cost C:

$$r_{L*} = -\frac{L}{\left(\partial L/\partial r_L\right)} + r_T + \left(\partial C/\partial r_L\right)$$

$$r_{D*} = -\frac{D}{\left(\partial D/\partial r_D\right)} + r_T(1-k) + \left(\partial C/\partial r_D\right)$$

Monopolistic bank

- Defining interest elasticity of demand for loans (ε_L) and deposits(ε_D) yields

$$\varepsilon_L = -\frac{\partial L/L}{\partial r_L/r_L} = -\frac{\left(\partial L/\partial r_L\right)}{L/r_L}$$

$$\varepsilon_D = \frac{\partial D/D}{\partial r_D/r_D} = \frac{\left(\partial D/\partial r_D\right)}{D/r_D}$$

Monopolistic bank

- With these elasticities, the profit maximizing r_{L*} and r_{D*} become

$$r_{L*} = \frac{r_T + \left(\partial C/\partial r_L\right)}{\left(1 - 1/\varepsilon_L\right)}$$

$$r_{D*} = \frac{(1-k) r_T + \left(\partial C/\partial r_D\right)}{\left(1 + 1/\varepsilon_D\right)}$$

Monopolistic bank

- Presenting this solution as equality between Lerner indices (price minus cost divided by price) and inverse elasticities

$$\frac{r_{L*} - r_T - \left(\partial C/\partial r_L\right)}{r_{L*}} = \frac{1}{\varepsilon_L\left(r_{L*}\right)}$$

$$\frac{(1-k) r_T - \left(\partial C/\partial r_D\right) - r_{D*}}{r_{D*}} = \frac{1}{\varepsilon_L\left(r_{D*}\right)}$$

Monopolistic bank

- A monopoly bank thus sets loans and deposits such that the price margin of loans and deposits over cost is equal to the inverse elasticity
- The greater the market power of the bank, the smaller the elasticity and the higher the Lerner index.
- In the limiting case of ε=0, these solutions become the ones of the competitive model derived before

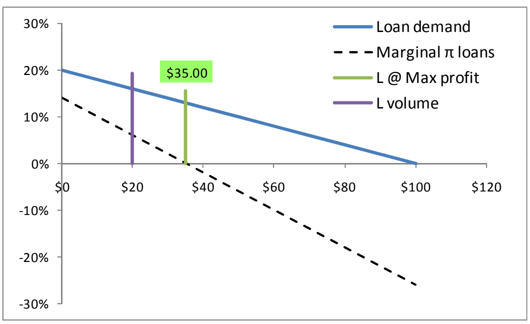
Monopolistic bank Numerical example

- Demand curve for loans linear and defined as r_L = slope (s) * L + a
  a: constant intercept, s: slope = (dr_L/dL)
- Management cost function defined as C=gamma$_L$*L (with constant gamma$_L$)
- Profit = (dr_L/dL)*L^2+(a-gammaL-r_T)*L
- Marginal profit = 2*(dr_L/dL)*L+(a-gamma$_L$-r_T)

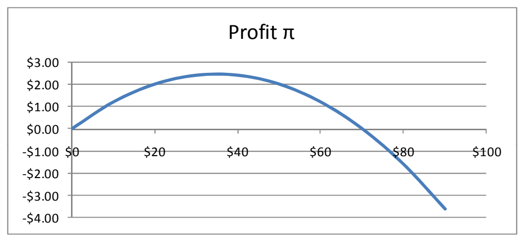
Numerical example

	A	B	C	D	E
1	**Monopolistic model (loans only)**				
2	**Loan demand curve parameters**				
3	Intercept rL(L=0)		20.00%		
4	Slope Ldemand (drL/dL)		-0.002	=-C3/100	
5	dL/drL		-500	=1/C4	
6	L(rL)	Loans	$ 20.00		
7	rL	Loan rate	16.00%	=C3+C4*C6	
8	εL	Elasticity	4.00	=-C5/(C6/C7)	
9	**Cost, revenue, profit**				
10	Riskfree rate rT		4.00%		
11	gammaL	constant dC/dL	2.00%		
12	C(L.D)	Mngt cost	$ 0.40	=C11*C6	
13	rT*L		$ 0.80	=C6*C10	
14					
15	R(L,rL)	Revenue	$ 3.20	=C7*C6	
16	Total cost		-$ 1.20	=-C6*C10-C12	
17					
18	π	Profit	$ 2.0000	=C15+C16	
19	dπ/dL	marginal π	$ 0.060	=2*C4*C6+C3-C10-C11	
20	**Max profit choice of L and rL**				
21	LOptimal (L*)		$ 35.00	=(C10+C11-C3)/2/C4	
22	rLOptimal (rL*)		13.00%	=C3+C4*C21	
23	εL @ L*	Elasticity @L*	1.86	=-C22/C21*C5	

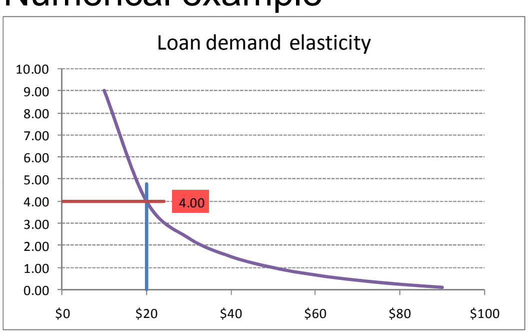
Numerical example



Numerical example



Numerical example



Oligopolistic version

- Monopolistic model clearly not realistic as banking industry not controlled by single institution
- We can, however, reinterpret Monti-Klein model as a model of imperfect (Cournot) competition with n banks each facing the same linear cost function $C_i = \gamma_D D_i + \gamma_L L_i$

with i = 1,2, …, n; constant γ_D and γ_D

Oligopolistic version

- Maximizing profit function of bank i where each bank set its deposits a D_{i*} = D/n, L_{i*} = L/n (D,L are deposit/loans of whole system), one finds

$$\frac{r_{L*} - r_T - \gamma_L}{r_{L*}} = \frac{1}{n\varepsilon_L(r_{L*})}$$

$$\frac{(1-k)r_T - \gamma_D - r_{D*}}{r_{D*}} = \frac{1}{n\varepsilon_L(r_{D*})}$$

Note: this solution is derived in Freixas & Rochet (1997, p.59/60)

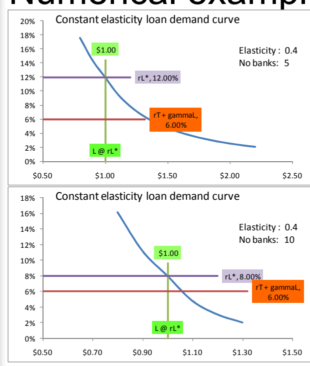
Oligopolistic version

- Only difference between monopoly case and the Cournot equilibrium is that elasticities are multiplied by n
- Can reinterpret as model of imperfect competition with two limiting cases:
  – n = 1: equivalent to monopolistic model
  – n=∞: perfect competition with fully elastic demand for loans / deposits

Numerical example

	A	B	C	D	E	F
1	**Oligopolistic version**					
2	**Const elasticity loan demand curve parameters**					
3	εL	Elasticity	0.40			
4	n	No of banks	5.00			
5	L0		$ 1.00	normalized for curve display		
6	delta L		$ 0.10	for curve display		
7	Riskfree rate		5.00%			
8	gammaL	constant dC/dL	1.00%			
9						
10	**Max profit choice of L and rL**					
11	rLOptimal (rL*)		12.00%	=(C7+C8)/(1-1/(C4*C3))		
12	LOptimal (L*)		$ 1.00	=C5		
13						
14	**Revenue, cost, profit**					
15	Revenue		0.12	=C12*C11		
16	C(L.D)	Mngt cost	(0.01)	=-C12*C8		
17	rT*L		(0.05)	=-C7*C12		
18	Profit		0.06	=SUM(C15:C17)		

Numerical example



Monti/Klein / Cournot criticism

- Monti/Klein provides some intuitive results that can be empirically tested but …
- While imperfections exist in markets, these are rarely related to restrictive practices of players (as assumed in this model)
- Imperfections are associated with incomplete or imperfect information, uncertainty, transaction costs

Examples applications Monti/Klein

- Assess impact of ceilings on deposit rates (Freixas & Rochet, 1997, p. 61)
  – If one assumes cost complementary of L and D, i.e. $\left(\frac{\partial^2 C}{\partial L \partial D} < 0\right)$ ceiling on deposit rates would have opposite effect (less loans offered, i.e. higher loan rates)
- Pass-through of official rate to loan and deposit rates (Bruggeman & Donnay, 2003)

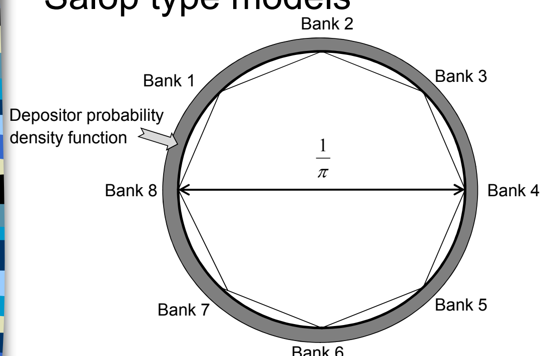
Double Bertrand competition

- Bertrand competition models as it is reasonable to assume that banks follow others' behaviour
- Double Bertrand competition because simultaneous competition on outputs (loans) and inputs (deposits).
- Some counter-intuitive results as no Walrasian equilibrium reached or no equilibrium at all.

Salop type models

- Location model of Salop (1979) is popular model of monopolistic competition
- Product differentiation is generated by 'transportation costs'
- The Salop model (simple version) assumes all n banks (i = 1,2,..,n) are located on a circle with a length normalized to one
- Customers (e.g. depositors) are uniformly distributed along same circle facing a proportional transportation cost to the closest bank (transportation cost factor α)

Salop type models



Salop type models

- Depositors being uniformly distributed, the optimal organisation of banking industry corresponds to symmetric location of n banks.
- The maximum distance travelled by depositor is $\frac{1}{2n}$

  and sum of all depositor transportation cost is

$$2n\int_0^{\frac{1}{2n}} \alpha x D dx = 2n \times \frac{1}{2}\alpha\left(\left(\frac{1}{2n}\right)^2 - 0^2\right) = \frac{\alpha D}{4n}$$

Salop type models

- With the unit cost of setting up a bank denoted by F, the optimal organisation of banking industry (optimal number of banks n*) is obtained by minimizing the sum of set-up and transportation cost:

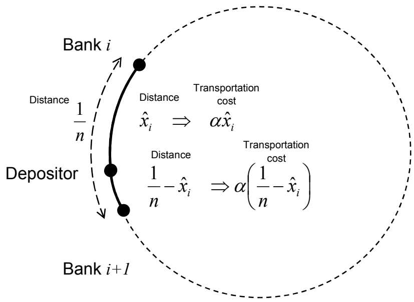
$$nF + \frac{\alpha D}{4n}$$

$$\partial\left(nF + \frac{\alpha D}{4n}\right) \Big/ \partial n = F - \frac{\alpha D}{4n^2} = 0 \Rightarrow n^* = \frac{1}{2}\sqrt{\frac{\alpha D}{F}}$$

Salop type models

- How many banks will appear if not entry restrictions, no rate regulations?
- Assume n banks enter simultaneously offering deposit rates $r_{D1}, r_{D2}, \ldots, r_{Dn}$
- To calculate the amount of deposit attracted by bank i (D_i), we compute location of marginal depositor, i.e. the depositor who is indifferent about going to bank i or i+1

Salop type models



Salop type models

- The distance $\hat{x}_i$ between marginal depositor and bank i is defined as

$$r_{Di} - \alpha\hat{x}_i = r_{Di+1} - \alpha\left(\frac{1}{n} - \hat{x}_i\right) \Rightarrow$$

$$\hat{x}_i = \frac{1}{2n} + \frac{r_{Di} - r_{Di+1}}{2\alpha}$$

- The total volume of deposits attracted by bank is thus

$$D_i = D\left[\frac{1}{n} + \frac{2r_{Di} - r_{Di-1} - r_{Di+1}}{2\alpha}\right]$$

Salop type models

- The profit of bank i is thus

$$\pi_i = D(r - r_{Di})\left[\frac{1}{n} + \frac{2r_{Di} - r_{Di-1} - r_{Di+1}}{2\alpha}\right]$$

  where r is rate at which will invest (lend) these deposits
- Chose r_{Di} which maximizes π_i for all i (all other rates are kept constant)

$$\frac{\partial \pi_i}{\partial r_{Di}} = D\left[(r - r_{Di})\frac{1}{\alpha} - \left(\frac{1}{n} + \frac{2r_{Di} - r_{Di-1} - r_{Di+1}}{2\alpha}\right)\right] = 0$$

Salop type models

- Chose r_{Di} which maximizes π_i for all i (continued)

$$\frac{\partial \pi_i}{\partial r_{Di}} = \frac{1}{\alpha}(r - r_{Di}) - \frac{1}{n} - \frac{2r_{Di} - r_{Di-1} - r_{Di+1}}{2\alpha} = 0 \Rightarrow$$

$$(r - r_{Di}) - \frac{\alpha}{n} - \frac{2r_{Di} - r_{Di-1} - r_{Di+1}}{2} = 0$$

- $r_{D1}, r_{D2} \ldots r_{Dn} = r - \alpha / n$ solves these n equations:

$$\left(r - r + \frac{\alpha}{n}\right) - \frac{\alpha}{n} - \frac{1}{2}\left(2r + 2\frac{\alpha}{n} - 2r + 2\frac{\alpha}{n}\right) = 0$$

Salop type models

- The profit for all banks thus becomes

$$\pi_1, \pi_2, \ldots \pi_n = \frac{\alpha D}{n^2}$$

- If we assume there are no restriction to entry, this profit is equal to setup costs F, the equilibrium number of banks n** is

$$n^{**} = \sqrt{\frac{\alpha D}{F}}$$

- This means free competition leads to too many banks: $n^{**} > n^{*}$