

**Fixed Income Research**

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**Convertible bond  
Winterthur Insurance  
with WinCAT coupons “Hail”**



**CHF 399 million subordinated 3-year convertible,  
conversion premium 20%**

**Knock-out of 2 1/4% annual coupon if any single hail  
or other storm damages more than 6,000 motor vehicles  
insured with Winterthur Insurance in Switzerland**

**Investment instrument to diversify portfolio since  
storm damage has practically no correlation to traditional  
financial market risk**

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## A new instrument on the CHF capital market

### Summary <sup>1</sup>

#### **Convertible bond ...**

**... with coupon payment dependent on major storm damage to insured vehicles (WinCAT coupons)**

**In the last 10 years two coupons would have been missed ...**

**... even taking into account the knock-out probability, the yield is higher than for a traditional convertible**

**Diversification outside traditional financial markets**

**Due to principal protection suitable for conservative investors**

The Swiss insurance company Winterthur Insurance (Winterthur) has launched a highly innovative product onto the Swiss franc capital market: a three-year subordinated 2<sup>1</sup>/<sub>4</sub>% convertible bond with WinCAT coupons (CAT = CATastrophe). This bond (face value CHF 4,700) may be converted into 5 Winterthur registered shares at maturity (European style option, conversion premium 20%). On the other hand the annual interest coupon of 2<sup>1</sup>/<sub>4</sub>% will only be paid out if during a particular year (observation period 1 November to 31 October) major storms do not damage more than 6,000 motor vehicles insured with Winterthur in any single event.

Had Winterthur launched an identical fixed interest convertible bond, the coupon rate would have been around 0.76% lower (approx. 1.49%). In other words, the investor receives an annual yield premium of 76 bp for bearing a small portion of Winterthur's damage to vehicles risk. This risk can best be assessed by examining a 10-year history of damage claims (see page 6). During this period a total of 17 events with more than 1,000 damaged vehicles were registered. Almost all these events were hail storms during the summer months (only two winter storms). However, in only two cases (hail storms) did the number of damaged vehicles exceed the coupon knock-out threshold of 6,000. Even if in future investors were to miss 2 in every 10 annual coupons, i.e. with a 20% knock-out probability, they would statistically still receive 80% of the 2<sup>1</sup>/<sub>4</sub>% WinCAT coupon. This results in an expected yield of 1.80%, or around 31 bp more than the 1.49% on a conventional fixed coupon convertible bond without catastrophe risk. A detailed calculation of the theoretical value of this special convertible bond using comprehensive statistical models is contained in the Analytical Annex to this brochure.

In the USA attempts to securitize catastrophe risk commenced some time ago. This bond offers a first-time investment opportunity to gain exposure to specific Swiss insurance risk. This risk transfer is very limited, however, and it is Winterthur's intention to test the Swiss capital market for this novel product category. Since catastrophe risks have very little correlation with performance on the traditional investment markets, they are particularly suited for portfolio diversification. A historical performance analysis shows that in fact higher yields would have been achieved with this instrument than with conventional convertible bonds. Only the coupon is subject to catastrophe risk, while the principal is fully redeemed at maturity. Note also Winterthur's excellent Moody's credit rating of Aaa (currently watchlisted, see <sup>2</sup> below).

<sup>1</sup> The legally binding terms of this bond may be found in the prospectus.

<sup>2</sup> Moody's Aaa credit rating for general unsecured debt for Winterthur is currently on the "watch list" for a possible downgrade. We expect a rating in the double-A range. Even in international comparison Winterthur will thus remain a premier quality insurer. Subordinated bonds are usually rated one notch lower.

## Why are catastrophe bonds issued?

### Investors: diversification

Increasing correlation between traditional investment markets caused by the globalisation of world economies is leading to fewer diversification opportunities for investors. Catastrophic events, however, have minimal correlation to any other investment market.

### Insurance: hedge against major catastrophe risk

From the perspective of the insurance industry, there is a noticeable shortfall between the claims potential of the largest US catastrophe risks (hurricanes in the south and east and earthquakes in the west) and the coverage capacity of the primary and re-insurance markets. For this reason work has been taking place in the USA to develop instruments which securitize insurance risk, in particular the peak risks of natural disasters. Insurance companies began to explore avenues of innovation after events such as Hurricane Andrew (damage in excess of USD 16 billion) caused the collapse of several property insurers and put considerable strain on reinsurance companies. In the USA alone around USD 20,000 billion is invested in the financial markets, of which around USD 7,000 billion in the equity markets. Total claims arising from Hurricane Andrew therefore only amount to 0.2% of the total market capitalisation of the US share market, which is within the normal daily fluctuation range of these markets.

### Financial markets can better absorb catastrophe claims

### USA: CBOT options on regional claims indices...

In 1992 the Chicago Board of Trade (CBOT) introduced futures and options on the Insurance Service Office (ISO) catastrophe indices which were followed in 1995 by options on the Property Claim Services (PCS) catastrophe indices. These options enable insurers to implement "call spread" strategies (buying a call option and simultaneously selling a call option with a higher exercise price), effectively providing them with a reinsurance layer.

Each option covers a so-called loss period (3 or 12 months), but the contract's cash settlement is only made 6 to 12 months after the end of the loss period. This time gap is required to register all claims and to make the necessary calculations to determine the index value. However, the liquidity of these options is currently rather low.

### CAT bonds: earthquake and hurricane bonds are planned ...

Various investment banks, particularly in the USA, are working on so-called CAT bonds. In contrast to the relatively illiquid catastrophe options which, as described, are mainly used by insurance companies for risk management, this type of instrument would also be suitable for other investors. Only a few rather small placements have been issued to date, most of them without explicit principal guarantee, i.e. capital can be lost entirely or partially in the event of a catastrophe. One example of this category would be the American International Group (AIG) notes offered late last year. According to the preliminary terms of these notes, investors will earn a maximal return of 25% if no relevant catastrophic event occurs. Yet they would lose 25% of their principal in the case of two or more damaging events. We believe that psychologically this loss of capital is a particular hindrance to the acceptance of the new instrument. While loss positions in the financial markets are not usually realised immediately and prices might reverse their trend, catastrophes will lead to definite, irrecoverable losses for CAT-bond investors.

### ... generally only a few issues

## Winterthur convertible bond with WinCAT coupons on Swiss hail risks

### Only the coupon is subject to storm risk

With its three-year 2 1/4% subordinated convertible bond with WinCAT coupons, Winterthur is offering investors a first-time opportunity of exposure to specific Swiss storm damage risk. It is important to note, however, that only the interest coupon will be subject to this risk element. It is knocked out if, in the course of a year, more than a fixed number motor vehicles insured with Winterthur in Switzerland are being damaged during any single major hail or other storm event. The bond may also be converted into Winterthur registered shares and, most importantly, the principal will be redeemed at 100% unless, of course, the holder opts to apply it towards conversion.

### Principal is guaranteed

### Hail and storm events resulting in over 6,000 vehicle damage claims mean coupon knock-out

Events leading to the knock-out of the interest coupon of 2 1/4% are defined<sup>4</sup> as follows: should Winterthur have to pay out damage claims on its comprehensive automobile insurance for over 6,000 vehicles as a result of at least one major hailstorm or any other storm during the period 1 November to 31 October the following year, then the corresponding annual coupon will not be paid out. Only vehicles damaged during any particular calendar day will count towards the knock-out threshold of 6,000. Thus should for example 5,000 motor vehicles be damaged by hail on each of two consecutive days in one summer, the investor would still receive the coupon for that particular year. A total of 773,600 motor vehicles<sup>5</sup> are currently insured with Winterthur in Switzerland. Should a significant quantitative change take place in this number, the knock-out limit of 6,000 damaged vehicles would be adjusted correspondingly.

### European-style conversion right Conversion premium 20%

At redemption the holder is entitled to convert the bond into Winterthur RS at an exercise price of CHF 940.- (European-style option). Given Winterthur's current stock market price, this would imply a conversion premium of approximately 20%.

## What is the fair price for this bond?

### Calculation of theoretical value of CAT bond in Analytical Annex

Since the analytical valuation for the new CAT bond is not trivial, we have added an Analytical Annex to this study. It contains a more quantitative approach to determining a theoretical value for this instrument.

### Historically 2 out of a potential 10 coupons would have been missed

Nevertheless, without advanced mathematics and merely based on the damage claim history, the investor can easily assess the effective risk<sup>6</sup> of this instrument (see table of damage claims on page 6). Since the beginning of 1987 a total of 17 storms were recorded where the number of damage claims of Winterthur-insured vehicles exceeded 1,000. Almost exclusively hail storms during the summer months were the cause of damage. In only two cases winter storms were involved, during which the number of vehicles damaged was significantly lower than 6,000. In only two cases (involving hailstorms), however, the number of damaged motor vehicles exceeded the knock-out threshold of 6,000. Thus, there would have been no coupon payment in 1992 and 1993 only. If in future the investor were to miss 2 in every 10 coupons (a probability of 20%), he would statistically still earn 80% of the 2 1/4% coupon, i.e. 1.80%.<sup>7</sup>

<sup>4</sup> Legally binding wording may be found in bond prospectus

<sup>5</sup> This number includes motor vehicles insured with Neuenburger Schweiz, Allgemeine Versicherungsgesellschaft (Neuenburger). Neuenburger and Winterthur intend to merge their operations as per 2 April, 1997.

<sup>6</sup> For further information and statistics on hail risk in Switzerland please refer to the brochure "Hagel und Schäden an Fahrzeugen" published by Winterthur Insurance (available in German only).

<sup>7</sup> With the number of coupon discrete and not continuous, actual yield will effectively be higher or lower (see Analytical Annex).

**Claim numbers of past events** (with over 1,000 damaged motor vehicles)

Year	Date	Event	No. of claims	Vehicles	
				insured index <sup>1</sup>	Adjusted claims <sup>1</sup>
1987	–		–	1.248	–
1988	–		–	1.204	–
1989	–		–	1.161	–
1990	27 Feb	Storm «Viviane»	1,646	1.127	1,855
	30 Jun	Hail	1,395		1,572
1991	23 Jun	Hail	1,333	1.104	1,472
	06 Jul	Hail	1,114		1,230
1992	21 Jul	Hail	8,798	1.098	<b>9,660</b>
	31 Jul	Hail	1,085		1,191
	20 Aug	Hail	1,253		1,376
	21 Aug	Hail	1,733		1,903
1993	05 Jul	Hail	6,589	1.099	<b>7,241</b>
1994	02 Jun	Hail	4,802	1.086	5,215
	24 Jun	Hail	940		1,021
	18 Jul	Hail	992		1,077
	06 Aug	Hail	2,460		2,672
	10 Aug	Hail	2,820		3,063
1995	26 Jan	Storm	1,167	1.067	1,245
	02 Jul	Hail	1,290		1,376
1996	20 Jun	Hail	1,262	1.000	1,262
<b>Total 17 damaging events</b>					

<sup>1</sup> Vehicles insured index

This index adjusts the number of claims of past events to current levels of insured vehicles. The index base of 1.000 for 1996 corresponds to around 773,600 insured motor vehicles. An index of 1.127 for 1990 thus means that a total of 686,400 motor vehicles were insured at that time. The claims figure of 1,855 for 27.2.1990 adjusted for the number of insured vehicles for 1996 is determined by multiplying the number of claims effectively recorded by the index for 1990. The index takes the integration of Neuenburger insurance company into account.

**Annual yield around 31 bp higher than for an identical bond without CAT risk**

The next step would be to determine the fair coupon for an identical 3-year convertible bond with conventional fixed coupon not dependent on catastrophe risk. According to generally known option pricing models, this type of bond would require a coupon of 1.49%. Even taking the knock-out probability of 20% into account, holders of this CAT bond would thus still receive a 31 bp yield premium compared to the conventional convertible.

**Control and public information**

**Independent audits**

The occurrence of an event upon which payment of the coupon is dependent is objective and can easily be established. Winterthur commissions independent audits of concrete numbers of claims arising out of relevant hail or storm events. Winterthur has a comprehensive quality control system and is the first and only insurance company in Switzerland

**Rapid information on relevant events**

to possess ISO<sup>8</sup>-9001 certification for its entire Swiss non-life business segment. This certification includes, in particular, all procedures related to claims processing, thus guaranteeing correct registration and categorizing of damage claims.

Furthermore, in accordance with the terms and conditions of this convertible bond, Winterthur is obliged to publish a statement on the estimated number of damaged motor vehicles no later than 14 calendar days after an event which could cause the claims limit to be exceeded. Winterthur must also make a public announcement that this limit has been exceeded no later than 10 calendar days after this has been determined. These measures are to ensure orderly and quick adjustment of the market price of this convertible.

**Winterthur shares: defensive stock with potential****Low risk insurance portfolio**

Winterthur is often criticized – unjustly – for its apparently low solvability ratio<sup>9</sup>. Of net premiums, 44% are in the life insurance business with its lower equity capital requirements. At the same time retail products such as motor vehicle insurance, property insurance and personal insurance, i.e. products for which statistical risks can more easily assessed, dominate the non-life segment. In contrast to Zurich Insurance, for example, major risk insurance (liability insurance, catastrophe risk insurance) is a less significant segment for Winterthur. Owing to the company's wide product range and geographic diversification, Winterthur shares are generally regarded as a defensive investment.

**Strong market position in retail insurance segment**

Competition in the retail property and automobile insurance segment is fierce. In the Swiss market deregulation has meant that Winterthur as market leader is also beginning to feel price pressure. As the largest insurance company, with its broad information base, Winterthur should nevertheless be well equipped to ensure appropriate pricing of its products in future. Equally, its size helps to lower distribution costs. With to its good market position, particularly also in Germany, Italy and Spain, we therefore believe that Winterthur will emerge from the current market "shake-out" as a much stronger company.

**Demographic trends indicate growth potential in life segment**

Growth potential for Winterthur is mainly in its life segment. The average age in western industrialized countries is rising and so is the demand for insurance. On top of this, there is a need to increasingly base pension systems on private insurance savings plans as state pensions funded by earnings contributions have clearly reached their capacity limits.

<sup>8</sup> ISO: International Standards Organization with headquarters in Geneva

<sup>9</sup> Solvability ratio = equity:net premiums ratio

## Historical performance

### Simulation

To study hypothetical ex-post performance of this new instrument, return, risk (standard deviation of return) as well as correlation to a conventional fixed coupon convertible was simulated. The simulation was conducted on a monthly basis for the period 28 February 1988 through 31 December 1996.

Simulation strategy:

- Buy: 3-year convertible with WinCAT coupons on 28 February
- Sell: after one year on 28 February (remaining time to maturity 2 years)
- Coupon: fixed based on 3-year zero-coupon swap rate on 28 February, adjusted by 25% (coupon knock-out probability used for pricing of this convertible). Seasonal patterns of hail storms in Switzerland were considered.
  
- Conversion price: to reflect a conversion premium of 21% at the coupon fixing date of 28 February

The following results were obtained:

- simple return = 5.96% p.a. (monthly basis)
- risk = 9.42% p.a. (monthly basis)

### Better performance than conventional convertible

With the same investment strategy, an identical fixed-coupon convertible would have achieved a return of 5.87% and a risk of 9.21%. As a result of a simulation period of a mere 9 years (historical swap rates dating back further could not be obtained), the historical knock-out probability climbed to 22% (from 20%). 1987, in which no damaging events occurred, would have been favourable for the CAT-bond but could thus not be considered. The new instrument nevertheless beats the conventional convertible in terms of return.

## Supplementary investment considerations

### Swiss law and pension funds

Swiss pension funds may under certain conditions include the new instrument in their portfolios even though the regulations do not explicitly mention this investment category. The German version of this report contains a more detailed explanation.

### Publication platform for storm and hail damage

Information on damage events relevant to this convertible bond with WinCAT coupons may be obtained from Winterthur Insurance's web site. This site will also include a list of historical events with more than 1,000 damaged motor vehicles.

Internet (World Wide Web): [www.winterthur.com](http://www.winterthur.com)

Information on events with damage figures exceeding 6,000 motor vehicles as well as any adjustments of this knock-out threshold will be published in

- Schweizerisches Handelsamtsblatt
- Neue Zürcher Zeitung
- Journal de Genève

### Taxation of WinCAT coupon

The WinCAT coupons will be subject to Swiss withholding tax ("Verrechnungssteuer"). According to Swiss tax law, the coupon will also be subject to income tax.

### Flat trading

This convertible with WinCAT coupons will be traded on a flat basis, i.e. accrued interest is included in the trading price. This is necessary since the coupon payment can only be assumed reasonably certain once the hail period has passed in September.

## Analytical Annex: Table of Contents

### **Determining the knock-out probability $P_{\text{CAT}}$**

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Probability of a coupon knock-out

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Present value of principal component

Value of WinCAT coupons

Value of conversion right

### **Probability distribution of the coupon knock-out**

Coupon knock-out probability distribution over the whole life span of the bond

## Determining the knock-out probability $P_{CAT}$

Hail and storm damage is modeled using a so-called composite Poisson distribution, i.e. the event frequency is approximated through a Poisson distribution and the size of an event through a Pareto distribution.  $P_{CAT}$  denotes the probability of at least one event with more than 6,000 damaged motor vehicles in one year .

### Poisson distribution to model the frequency of events

The probability function of the Poisson distribution in its general form:

$$Poisson_{\lambda}(n) = \exp(-\lambda) \cdot \frac{\lambda^n}{n!}$$

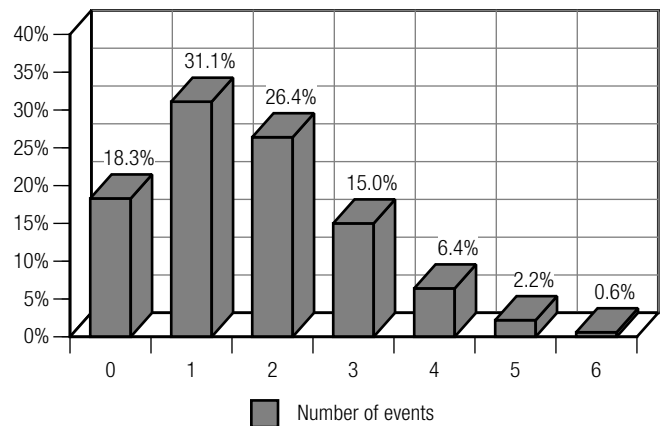
- $\lambda$ : is the average number of events expected to occur annually
- $Poisson_{\lambda}(n)$ : is the probability of observing exactly  $n$  events within one year given a Poisson parameter of  $\lambda$  .

The history of damage events (see table p. 12) implies an annual average of 1.7 events with more than 1,000 damaged motor vehicles. Modeling the probability of these events with the Poisson distribution, yields an estimated value  $\hat{\lambda}_{1,000} = 1.7$  for the Poisson parameter  $\lambda$ . The illustration to the right shows the probability that a given number of events will occur. For example, there is a 31.1% probability of precisely one event occurring resulting in more than 1,000 claims.

Theoretically it would equally be possible to derive the frequency of events involving more than 6,000 damaged motor vehicles from historical observations. Determining  $\lambda_{6,000}$  simply by summing the number of actually observed events would

give an estimated value  $\hat{\lambda}_{6,000} = 0.2$ . However, since this value is determined based on only two events observed, it has very little statistical significance.

Probabilities Poisson distribution  $\lambda_{1000} = 1.7$



In fact, the standard error of the estimate (standard deviation) amounts to

$$\hat{\sigma}(\hat{\lambda}_{6000}) = \sqrt{\hat{\lambda}_{6000} / \text{No. of years}} = 0.14$$

and the variation coefficient (standard error of the estimate divided by the expected value) is around 70%.

Hence for a statistically more accurate method a more significant  $\hat{\lambda}_{6,000}$  must be determined by multiplying the already known  $\hat{\lambda}_{1,000}$  by the probability that more than 6,000 motor vehicles are damaged in a single event. This probability is calculated using the Pareto distribution as outlined on p.13.

**History of damage events** (calculated on basis of no. of vehicles in 1996)

Year	Date	Event	Claims > 1,000	Claims > 6,000
<b>1987</b>	–		–	–
<b>1988</b>	–		–	–
<b>1989</b>	–		–	–
<b>1990</b>	27 Feb	Storm «Viviane»	1,855	–
	30 Jun	Hail	1,572	–
<b>1991</b>	23 Jun	Hail	1,472	–
	06 Jul	Hail	1,230	–
<b>1992</b>	21 Jul	Hail	9,660	9,660
	31 Jul	Hail	1,191	–
	20 Aug	Hail	1,376	–
	21 Aug	Hail	1,903	–
<b>1993</b>	05 Jul	Hail	7,241	7,241
<b>1994</b>	02 Jun	Hail	5,215	–
	24 Jun	Hail	1,021	–
	18 Jul	Hail	1,077	–
	06 Aug	Hail	2,672	–
	10 Aug	Hail	3,063	–
<b>1995</b>	26 Jan	Hail	1,245	–
	2 Jul	Hail	1,376	–
<b>1996</b>	20 Jun	Hail	1,262	–
No. of years (87–96)			10	10
No. of events			17	2
Average annual number of events observed (event frequency)			$\hat{\lambda}_{1000} = 1.7$	$\hat{\lambda}_{6000} = 0.2$
Standard error of event frequency ( $\sqrt{\hat{\lambda} / \text{No. of years}}$ )			$\hat{\sigma}(\hat{\lambda}_{1000}) = 0.41$	$\hat{\sigma}(\hat{\lambda}_{6000}) = 0.14$
Variation coefficient ( $\hat{\sigma} / \hat{\lambda}$ )			24%	70%

**Pareto distribution to model the size of an event (number of claims per event)**

The Pareto probability distribution is used to model the conditional probability that in an event with more than  $a$  damaged vehicles the number of damaged vehicles does not exceed  $x$ .

In its general form the function is

$$\begin{aligned} \text{Pareto}_{a,b}(x) &= 1 - \left(\frac{a}{x}\right)^b \text{ if } (x > a) \\ \text{Pareto}_{a,b}(x) &= 0 \text{ if } (x \leq a) \end{aligned}$$

where

$x$ : the number of damaged motor vehicles per event.

Parameters:

$a = 1,000$  Minimum claim: The historical database includes only events with more than 1,000 damaged motor vehicles; i.e. by definition at least 1,000 motor vehicles are damaged in any single event.

$\hat{b} = 1.371$  Maximum likelihood estimate of  $b$  multiplied by factor  $(n - 1)/n$ , so that  $\hat{b}$  is unbiased

$$\hat{b} = \frac{n-1}{\sum_{i=1}^n \ln\left(\frac{x_i}{a}\right)}$$

$n$ : Number of events observed ( $n = 17$ )

$x_i$ : Number of motor vehicles damaged in  $i$ -th event

A statement can also be made as to the accuracy of the estimated value  $\hat{b}$ . Its variance is given by  $\text{Var}(\hat{b}) = b^2 / (n - 2)$ ;

therefore

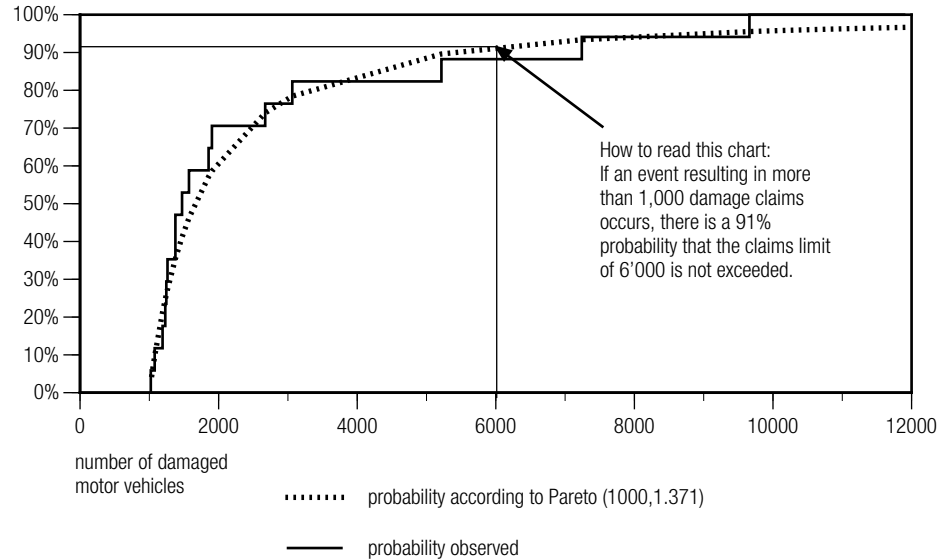
$$\hat{\sigma}(\hat{b}) = \hat{b} / \sqrt{n-2} = 0.35 \text{ (standard error of the estimate } \hat{b})$$

**Estimating the number of events with more than 6,000 claims in a particular year ( $\hat{\lambda}_{6,000}$ )**

According to the first paragraph, each year 1.7 events with more than 1,000 damaged motor vehicles may be expected ( $\hat{\lambda}_{1,000} = 1.7$ ). The Pareto distribution with the above parameters  $a$  and  $\hat{b}$  for  $x = 6,000$  yields a value of 91.43%; i.e. there is a 91.43% probability that less than 6,000 motor vehicles will be damaged in an event in which more than 1,000 motor vehicles are damaged. There is therefore only an 8.57% (100% - 91.43%) probability that the claims limit of more than 6,000 motor vehicles damaged will be exceeded. This yields  $\hat{\lambda}_{6,000} = \hat{\lambda}_{1,000} \cdot (1 - \text{Pareto}_{1,000,1.371}(6,000)) = 0.146$

The value of 0.146 is somewhat lower than the historically observed average annual frequency of 0.20, although, as already noted, this figure is not statistically significant.

**Distribution of number of damaged motor vehicles (in events with more than 1,000 damaged motor vehicles)**



**Probability of a coupon knock-out**

The coupon is not paid when the claims limit of 6,000 is exceeded one or more times during the observation period:

$$P_{CAT} = P(N \geq 1) = 1 - P(N = 0),$$

whereby  $N \approx$  Poisson-distributed with  $\lambda = \lambda_{6,000}$   
and therefore

$$P_{CAT} = 1 - \exp(-\lambda_{6,000}).$$

As  $\lambda_{6,000} = \lambda_{1,000} \cdot (1,000/6,000)^b$ ,  $P_{CAT}$  may also be expressed as

$$P_{CAT} = 1 - \exp(-\lambda_{1,000} \cdot 6^{-b}).$$

Based on the mathematical model and the parameters determined above

$$\hat{P}_{CAT} = 1 - \exp(-\hat{\lambda}_{6,000}) = 13.56\%.$$

According to this estimate, an event with more than 6,000 vehicles damaged is to be expected on average every 7 years. This compares to the historically observed frequency over the last 10 years, when such events occurred on average every 5 years. The above probability based on the mathematical model would thus appear to be too low. However, as already noted, the historically observed frequency in the case of only 2 events is not statistically significant.

### Standard error of $P_{CAT}$ -estimate

Nevertheless, the above estimate according to the mathematical model also involves some statistical uncertainty. The standard error of the estimate  $\hat{P}_{CAT}$  (standard deviation) can be estimated from the following expression (first order Taylor expansion):

$$\sigma^2(\hat{P}_{CAT}) = \left(\frac{\partial P_{CAT}}{\partial \lambda_{1000}}\right)^2 \cdot \sigma^2(\hat{\lambda}_{1000}) + \left(\frac{\partial P_{CAT}}{\partial b}\right)^2 \cdot \sigma^2(\hat{b})$$

and substituting with the values found earlier

$$\hat{\sigma}(\hat{P}_{CAT}) = 8.6\%.$$

In view of the large standard error of the estimate and the fact that the observed frequency of 20% lies within the interval  $\hat{P}_{CAT}$  (13.56%) to  $\hat{P}_{CAT} + \hat{\sigma}$  (22.16%), the standard error of the estimate is added to the knock-out probability calculated according to the actuarial method. This obviously implies a risk premium.

From the financial market standpoint, however, it is primarily historical empirical values which are of interest. During the last 10 years, such events occurred every 5 years (knock-out probability = 20%).

In order to take account of both the insurance and financial market perspective, a knock-out probability  $P_{CAT}$  of 25% was applied to set the coupon of this bond. The investor thus receives a risk premium compared to the results of both models.

# Calculating the theoretical value of the convertible bond with WinCAT coupons

This chapter describes the calculation of a theoretical value of the convertible bond with WinCAT coupons applying mathematical models. The analysis becomes more palpable by separating the evaluation of individual value components :

- The present value of the principal that either becomes due at maturity or is applied towards conversion into shares
- The present value of the expected coupon payments (i.e. weighted by the knock-out probability)
- The value of the conversion right

## Theoretical value of convertible with WinCAT coupons

**Present value of principal**

$$\frac{\text{par value}}{(1 + r_t)^t}$$

+

**Present value of expected coupon payments**

$$\sum_{i=1}^t \frac{(1 - P_{CAT}) \cdot \text{coupon}}{(1 + r_i)^i}$$

+

**Conversion right**

$$P_{CAT} \cdot \text{europ. call-option}$$

(exercise price = face value)

+

$$(1 - P_{CAT}) \cdot \text{europ. call option}$$

(exercise price = face value + last coupon)

$P_{CAT}$  : Coupon knock-out probability

$r_i$  : Zero-coupon rate in ith year

$t$  : Years to maturity (in this case 3 years)

### Assumptions regarding interest-rate structure

The discount rates correspond to the zero-coupon yield on Confederation bonds plus a spread of 35 bp.

	Discount rate	Discount factor
1st year	1.87%	0.9816
2nd year	2.33%	0.9550
3rd year	2.57%	0.9267

### Present value of the principal

The value of the principal component is the present value of the amount repayable at maturity:

$$\text{Principal component: } 0.9267 \cdot \text{CHF } 4,700.- = \text{CHF } 4,355.49 \quad (= 92.67\% \text{ of face value})$$

### Value of the WinCAT coupons

The value of all WinCAT coupons is the sum of the present values of expected coupon payments, whereby the expected coupon is calculated on the basis of a 25% knock-out probability, i.e. with a risk premium.

1. Coupon: $0.9816 \cdot 75\% \cdot 2 \frac{1}{4}\% \cdot \text{CHF } 4,700.- =$	CHF 77.85
2. Coupon: $0.9550 \cdot 75\% \cdot 2 \frac{1}{4}\% \cdot \text{CHF } 4,700.- =$	CHF 75.74
3. Coupon: $0.9267 \cdot 75\% \cdot 2 \frac{1}{4}\% \cdot \text{CHF } 4,700.- =$	CHF 73.50
<b>Present value of all coupons:</b>	<b>CHF 227.09</b> <span style="float: right;"><b>(= 4.83% of face value)</b></span>

## Theoretical value of the conversion right

### Assumptions:

Current share price:	CHF 783.–
Volatility:	17%
Dividend payments:	CHF 21.– (on 25 June)
Risk-free interest rate:	2.20%
Conversion right:	one bond (CHF 4,700.- face value) converts into 5 shares

The value of the conversion right is the sum of 2 European call options with different exercise prices, weighted by knock-out probability  $P_{\text{CAT}}$ . Since on payment of the last coupon (probability = 80%) the investors foregoes this coupon on conversion, the exercise price corresponds to the face value plus this coupon payment. If the last coupon is knocked out (probability 20%), the exercise price equals the face value.

Probability	Exercise price	Value of call option
80%	CHF 4,805.75	CHF 154.95
20%	CHF 4,700.—	CHF 174.30
<b>Weighted sum</b>		<b>CHF 158.82</b> (= 3.38% of face value)

<sup>1</sup> In order to keep the valuation as conservative as possible, the historical coupon knock-out probability of 20% instead of 25% used elsewhere in this calculation was applied here.

## Theoretical total value

Principal	CHF 4,355.49	92.67%
WinCAT coupons	CHF 227.09	4.83%
Conversion right	CHF 158.82	3.38%
<b>Theoretical total value</b>	<b>CHF 4,741.40</b>	<b>100.88%</b>

This valuation method is valid only for the time of issue, since the probability of a coupon knock-out decreases once the main hail season in the summer has passed. Thus the knock-out probability for the current coupon must be continually adjusted so that it remains close to the normal value of 25% until the end of May and then decays towards 0% by the end of August.

## Probability distribution of the coupon knock-out

The calculation of the coupon knock-out probability for the total life of the bond is based on the so-called binomial distribution. This distribution represents the probability that over the 3-year bond life  $k$  coupons are paid out, whereby the knock-out probability for one year individually amounts to 20%:

$$\text{Binomial distribution}(k) = \frac{3!}{k! (3-k)!} \cdot 0.2^{3-k} \cdot (1-0.2)^k$$

Number of coupons paid (k)	Probability of			Yield		
	exactly k coupons	at least k coupons	paid	absolute	weighted	
3	$0.8^3$	=	51.20%	51.20%	2.25%	1.15%
2	$3 \cdot (0.8^2 \cdot 0.2)$	=	38.40%	89.60%	1.50%	0.58%
1	$3 \cdot (0.8 \cdot 0.2^2)$	=	9.60%	99.20%	0.75%	0.07%
0	$0.2^3$	=	0.80%	100.00%	0.00%	0.00%
<b>Total</b>			<b>100%</b>			<b>1.80%</b>

A weighted average yield of 1.80% is obviously purely theoretical since this would imply a knock-out of 2.4 coupon payments (=80% • 3 coupons).

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